WEIBULL PROBABILISTIC APPROACH IN A GENERALIZED COMPOSITE MATERIAL APPLICATION TO FLEXURE TEST

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Abstract

In accord with the Weibull's theory of the probabilistic strength a generalized composite material model constituted by several materials is given. Such model taking into account that fracture process can occur in volume, interface and internal surface of composite material. From a theoretical point of view a general equation, developed here, which describes the process of total rupture of a composite material, allows us to generate a model that can be numerically treated. After that we introduce an equation that allows to determining the place where the process of rupture starts, i.e. the local probability. In order to show the applicability of model a rectangular beam subjected to flexure was treated. Then, employing a Weibull´s functions of three – parameters the cumulative and local probabilities functions for a rectangular beam were determined. Due to the complex behaviour of a general composite material in relation to strength we give a numerical simulation procedure to obtain cumulative and local probabilities of fracture or yielding, depending if the material is brittle or ductile or if have both kind of components. By means of a simulation procedure two series of random numbers are necessary to describe the process of fracture or plastic deformation of a composite material. The first series of random numbers allows us to determine the stress of rupture or the transition from linear to non-linear deformation or yielding and the second series gives the localization where it is produced. Starting with the knowledge of Weibull´s parameters obtained by experimental test, the simulation procedure applied to a rectangular beam allows us to determining fracture strength and his respective fracture location. Finally, it is possible to observe the simulation processes fitness to the experimental results plotting Weibull´s diagrams.

1. Introduction

In a composite material or in a complex structure of brittle materials, the process of fracture is initiated by the propagation of the first crack and continues with the stress redistribution having many complexities. In general, only those cases in which, after the successive breakings, the stress field can be determined in a simple form as have been treated [1]. If the system is not brittle or if it is only partially, the problem becomes complex again. In this case, only de simple instances have been treated too [2]. For example, the simplest case of a composite system is the case of a system of parallel brittle bars and it was partially studied by Kittl and Díaz [3] and more recently by Martínez [4] in a generalized form. On the other hand, if the composite system is constituted only by ductile elements, σ_f being the stress when the plastic determination begins, the event can be treated like a problem where the elements are fractured just at the stress σ_f considering that after reaching σ_f, an increase of deformation does not mean an increase of tension or, in other words, the material does not presents work-hardening or it is an elastic-perfectly plastic material. Moreover, as assuming plastic deformation of a given ductile element after reaching the stress σ_f, then it is possible to suppose that this one no longer works when the material reaches the deformation corresponding to the maximal necking and when happening this the respective element can be removed from the analysis, as can be done with a brittle element. The other problem is that, normally, one deals with composite materials formed by a matrix containing a disperse phase, as for example filaments, which have a random distribution and, hence, they have to be considered from the statistical standpoint. The aim for this work is to provide a general approach which describes the cumulative and local probabilities respect to the fracture, yielding or plastic deformation of generalized composite materials or complex structures and applying to rectangular beam subjected to flexure employing a simulation process.

2. Cumulative probability of fracture

The general equation of cumulative probability of fracture or yielding [2,5,6], $F(\sigma)$, of a composite material which corresponds to a body composed of N different materials distributed in V_i, volumes, S_{ij} interfaces and S_{i0} external surfaces, subjected to some uniaxial stress field σ (r) $\leq \sigma$, is:

$$\begin{aligned}
&F(\sigma)=1-\exp\{-\xi(\sigma)\} \quad ; \ \sigma_n=\sigma_n(\sigma) \quad ; \ \tau=\tau(\sigma) \\
&\xi(\sigma)=\frac{1}{V_0}\sum_i\int_{V_i}\phi_i^V[\sigma(r)]dV_i+\frac{1}{S_0}\sum_{i,j}\int_{S_{ij}=V_i\cap V_j\neq 0}\phi_{ij}^{\tau}[\tau(r)]dS_{ij} \\
&\quad+\frac{1}{S_0}\sum_{i,j}\int_{S_{ij}=V_i\cap V_j\neq 0}\phi_{ij}^{n}[\sigma_n(r)]dS_{ij}+\frac{1}{S_0}\sum_i\int_{S_{i0}}\phi_{i0}^{n}[\sigma_n(r)]dS_{i0} \\
&\quad+\frac{1}{S_0}\sum_i\int_{S_{i0}}\phi_{i0}^{\tau}[\tau(r)]dS_{i0} \quad ; \ i,j=1,2,...,N
\end{aligned} \tag{1}$$

where ϕ_i^V is the specific risk of volume fracture function for the material i, ϕ_{ij}^{τ} is the specific risk of surface fracture function of the interface between the material i and the material j subjected to a tangential stress τ, ϕ_{ij}^{n} is the specific risk of surface fracture function of the interface ij subjected to a normal stress σ_n, which is function of the maximum stress σ, ϕ_{i0}^n is the specific risk of surface fracture function of the external surface S_{i0} subjected to a normal stress σ_n, ϕ_{i0}^τ is the specific risk of surface fracture function of the external surface S_{i0} subjected to a tangential stress τ, r is the position vector, V_0 is the unit volume and S_0 is the unit surface. The equation (1) can be easily extended to the case of a triaxial stress field.

3. Local probability and location of the first fracture

For the case of composite materials, the location where the first fracture appears is very important because this fracture propagates until one or more of the composite parts are broken or even until the system reaches the total collapse. In every stage, after a given crack propagation, and owing to the applied stress is being gradually increase, the stress field is changed and equation (1) has to be successively applied until, finally, such composite material no longer resists. As was mentioned above, this can easily be made in the simplest cases, as for example in a system of parallel bars, solved partially by Kittl and Díaz [3] and completely by Martínez [4], where the composite is considered at formed by only one phase subdivided and which can be assimilated to a rope, the oldest composite. However, in the general case the treatment is quite complex. There is a controversy about the formalism that must be employed to treat the local probability of fracture and was discussed previously [7]. Here we use the formalism developed by Kittl and Camilo [8]:

$$\frac{1}{n}\frac{dn(r)}{dV} = \frac{\phi\left[\sigma(r)\right]}{\int_V \phi\left[\sigma(r)\right]dV} \tag{2}$$

where n is the total number of failed bodies and dn(r)/dV those which began to fail within the volume dV located in r. Equation (2) gives the fraction failures per unit volume in the point r of the body at the stress σ.

4. Application to a rectangular beam subjected to flexure

In order to simplify the analysis, without loss generality, we consider a rectangular beam of length L, height h and width b subjected to flexure by means a load fracture P located in the middle span, i. e., three point bending test. We assume the fractures occur in the face of the beam opposite to the applied load. In such case and employing a Weibull´s function of three – parameters m, σ_0 and σ_L, the cumulative probability of fracture or yielding, in accord with equation (1), is:

$$F(\sigma) = 1 - \exp\left\{ -\frac{bL}{S_0(m+1)}\left(\frac{\sigma_L}{\sigma_0}\right)^m \frac{\sigma_L}{\sigma}\left(\frac{\sigma}{\sigma_L} - 1\right)^{m+1} \right\} \tag{3}$$

$$\sigma_L \le \sigma \le \infty$$

where m and σ_0 are the Weibull parameters depending on the manufacture process of the material and σ_L is the lower limit stress under which there is no fracture. On the other hand, the local probability of fracture or yielding, in accord with equation (2) is:

$$\begin{cases} \dfrac{n(x)}{n} = \left(\dfrac{\dfrac{2x}{L}\dfrac{\sigma}{\sigma_L} - 1}{\dfrac{\sigma}{\sigma_L} - 1} \right)^m & ; \quad \sigma \to \infty \quad \dfrac{n(x)}{n} = \left(\dfrac{2x}{L} \right)^{m+1} \\ \sigma \geq \sigma_L \quad ; \quad x \geq \dfrac{L}{2}\dfrac{\sigma_L}{\sigma} & \end{cases} \tag{4}$$

where x is the axial coordinate measure from the fracture location to nearest bearing. Due to the complex behaviour of a general composite material in relation to strength we give here a simulation procedure to obtain cumulative and local probabilities of fracture or yielding, depending if the material is brittle or ductile of if have both kind of components. The basis of simulation is that all cumulative probabilities are equally probable. The procedure starts with the knowledge of Weibull´s parameters m, σ_0 and σ_L from an experimental test for some material subjected to three point bending test. Using a random number generation function we can take a double series of random numbers $\{0 \leq \lambda_i^j \leq 1; 0 \leq \mu_i^j \leq 1\}$ where i = 1, 2, …, N and j = 1, 2, …, M then it is possible to simulate the fracture strength and his respective location. With the first series of random number λ_i^j we can obtain a set of simulated strength values $\{\sigma_1^j, \sigma_2^j, \ldots, \sigma_N^j\}$using λ_i^j equal to cumulative probability in equation (3). On the other hand, the second series of random numbers μ_i^j allows to determining fracture location $x_i^j = (L/2)\ \mu_i^j$ which corresponds to fracture strength σ_i^j. If we applied this procedure N times, for each fracture strength and with M iterations, we obtain M strength of fracture $\{\sigma_1^1, \sigma_1^2, \ldots, \sigma_1^M\}$ and his respective M fracture locations $\{x_1^1, x_1^2, \ldots, x_1^M\}$. After that with both values set we can plot his respective Weibull´s diagram, cumulative and local probabilities, employing an estimator to evaluate such probabilities. The diagrams must be compared, respectively, with equations (3) and (4) and applying, for example, a chi – square test in order to see the simulation processes fitness to the experimental results.

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